

❖ *Quantifiers* ❖

5.4. Quantifiers and Variables

Quantifiers such as “all” and “some” are our last new bit of logical form. But since quantified sentences will turn out to be the most complex examples of logical form so far, we begin some English examples to sharpen our understanding of what quantifiers are – and what they’re not.

1. What Quantifiers Are Not: Names Revisited. Having mastered translation of proper names into the formal language, it’s tempting to sweep other phrases under the umbrella of proper names by translating them the same way. The following two sentences, for instance, seem nicely parallel.¹

- (1) **Elements of Logic** is a boring book.
- (2) **Something** is a boring book.

Since “something” fills the grammatical subject position of (2) just as the proper name “Elements of Logic” does in (1), it could seem sensible to translate “something” by a name letter.

But the parallel breaks down on further analysis.

- (1) Elements of Logic is a boring book.
- (3) Elements of Logic is boring, and Elements of Logic is a book.
- (2) Something is a boring book.
- (4) Something is boring, and something is a book.

With proper names we made a practice of translating ‘stacked up’ predicates – like “is a boring book” – as a conjunction in disguise. That treats (1) and (3) as equivalent in logical meaning. And that looks like the right result: (1) and (3) mean the same thing; and whenever one of these sentences is true the other is as well.

¹ Adapting an example from (Quine 1959: 83-84).

But (2) and (4) aren't true in all the same situations.

- (2) Something is a boring book.
- (4) Something is boring, and something is a book.

In a situation containing just two objects – a boring speaker and an interesting book – (4) would be true but (2) would be false.

If that seems too subtle, the next two sentences draw the same distinction more boldly.

- (5) Something both is and isn't cat.
- (6) Something is a cat, and something isn't a cat.

What (6) reports certainly seems possible – indeed, the actual world is a situation where something's a cat (say, Neko) and something isn't a cat (say, Rex). But (5) looks like a flat-out contradiction, false in every possible situation (including the actual one). Hence the actual world illustrates that **(5) and (6) aren't true in all the same situations**; and, predictably, the two sentences **don't mean the same thing**.

Replacing “something” with proper name “Rex” yields (7) and (8), which do seem logically equivalent. (They **both** look like contradictions.)

- (7) Rex both is and isn't cat.
- (8) Rex is a cat, and Rex isn't a cat.

This illustrates that the quantifier “something,” even when appearing in the subject position of an English sentence, doesn't behave like a proper name. And that's why we resist translating English quantifiers as name letters.

(We could make the same case in terms of truth and validity: given the obvious importance of truth and falsehood to validity, and the clear differences in truth and falsehood between Sentences (5) and (6), our formal language had better have a way of showing the difference between such sentences – when a sentence is talking about the same object(s) throughout, like (5), and when not, like (6). But as long as we translate quantifiers as names, that difference will be papered over – as (7) and (8) make clear.)

2. Quantifiers and Variables. Further examples steer us toward a better translation of sentences with quantifiers. Suppose, first, that a materialist philosopher of mind makes the following **universal** claim.

(9) **Everything is physical.**

The predicate “is physical” is translated by a predicate letter.

G: ___ is physical

We still lack a way of talking formally about **every** object. But a long-winded rephrasing of Sentence (9) offers a clue.

(10) **For every object, the following holds of it:**
it is physical.

Put this way, quantified sentence (9) breaks into two parts: (i) a quantifying phrase announcing that something holds of every object, and (ii) the mini-sentence “it is physical” on the right, saying something about those object(s).

The mini-sentence “it is physical” resembles the familiar sentences such as “Rex is physical” which pose no obstacle to translation. And in some ways “it” acts like the proper name “Rex”: both words can fill the blank in a predicate, yielding a sentence of English; and both serve to refer to some object. Here again we may be tempted to translate the term as a proper name.

But there’s an important difference as well. The name “Rex” is a **proper** name precisely because it always points to the same object. By contrast, the word “it” can refer to different things from one sentence to the next. I can point to a log cabin and say “It is made of wood,” then point to the Cathedral of Learning and say “It isn’t made of wood”.² In so doing I’ve uttered no contradiction – since “it” in the two sentences pointed at different things. By contrast, if I say “Suki is from Pennsylvania” and “Suki isn’t from Pennsylvania” I have contradicted myself – since the proper name “Suki” refers consistently to the same object.

² Indeed, even within the same sentence “it” can refer to different things: if I say “It is made of wood but it isn’t,” I’ve uttered no contradiction if I pointed to a log cabin while uttering the first “it” but pointed to a stone cathedral while uttering the second “it”.

Because “it” can **vary** in what it points to, the word refers successfully only with outside help – a pointing finger, or a context where one especially point-worthy object looms large. By varying the context or target of the finger, we vary the reference of “it”. By contrast, the proper name “Suki” refers to Suki whether or not I’m pointing to her or looking at her.

For that reason an “it” sentence of English without such outside help **fails to make a complete claim**. If you find a scrap of paper with the sentence “It is from Pennsylvania” written on it, you don’t know what claim is being made – not even if you know the geographical origin of every object in the world. But a scrap of paper reading “Suki is from Pennsylvania” on its own expresses a complete sentence, capable of truth or falsehood. (In fact it’s false.) An “it” sentence isn’t the sort of complete-claim-maker we find in a sentence using a proper name.

To highlight those differences, we resist translating “it” (and related terms like “this” and “that”) as name letters. Instead we add new pointing symbols to the formal language: lower-case letters “p” through “z”. These are the **variables**.³

Variables: lower-case letters “p” through “z”

In terms of sentence construction, variables show up in the same locations as proper names: after a predicate letter. Because name letters and variables serve the semantic role of referring to objects, we view them as different species within a larger family – what we’ll call “terms”.

Term: any name letter or variable

So using the previous translation key, we translate the ambiguous English mini-sentence “it is physical” as follows.

G: ___ is physical

It is physical.

Gx

³ Just as we had a variety of sentence letters at our disposal in previous chapters, but followed logical tradition in beginning with “P” and “Q,” we follow tradition here in **starting with variables “x,” “y,” and “z,”** in our formal translations – resorting to other variables only in the rare case where we need more.

Variables and predicate letters thus take our original English sentence part-way into formal translation.

For every object, the following holds of it:

it is physical

For every object, x, the following holds of x: Gx

And note: in English the variable term “it” appears even in the quantifying left part of the sentence: “For every object, the following holds of **it**”.

We then need a symbol for universal terms like “every” and “all”. We’ll express these formally by the **universal symbol** “ \forall ”. (When reading this symbol aloud, it’s read “universal”.)

In formal translation we then combine the universal symbol with a variable to yields a **universal quantifier**. So “ $\forall x$ ” means “for every object, x, the following holds of x”.

Now we can translate the entire English sentence.

G: __ is physical

(9) **Everything is physical.**

(For every object, x, the following holds of x: x is physical)

(11) **$\forall x Gx$**

Next consider a weaker claim, which even a dualist philosopher of mind could agree to.

(12) **Something is physical.**

Rephrasing that claim in the same wordy fashion as before yields the following.

(12) Something is physical

For some object, the following holds of it:

it is physical

Employing the same translation key, the translation begins like this.

G: __ is physical

For some object, x , the following holds of x : Gx

As a formal means of expressing “some,” we introduce the **existential symbol** “ \exists ” (pronounced “existential”). Like the universal, the existential symbol combines with a variable to make an **existential quantifier**.

We then translate the whole sentence like so.

(12) Something is physical

(13) $\exists x Gx$

Finally, suppose an idealist philosopher of mind makes the following claim.

(14) **Nothing is physical.**

It might seem we’d need a third quantifier here to translate “nothing”. But while we could add that to the formal language, it turns out we can translate this sentence into the formal language using just symbols on hand.

For intuitively, this claim contradicts the previous claim that “something is physical”. The claim that “Nothing is physical” is true just where it’s false that “something is physical”; and the claim that “something is physical” is true just where it’s false that “nothing is physical”.

So a “nothing” sentence can be read as the **denial of an existential sentence**.

G: ___ is physical

Nothing is physical

(15) $\sim \exists x Gx$

Interestingly, this isn’t the only way to express “nothing” in the (expanded) formal language. Indeed, we’ll later find different, but semantically equivalent, ways of expressing all the quantified sentences covered here. But we now have at least one way to translate each sort of quantified sentence of English into the formal language of Chapter Five.

Summary: Quantifiers and Variables

- **Variables:** lower case letters **p through z** (with or without numerical subscripts). Variables are the formal counterpart to short-term, context-sensitive pointers of English such as “it”. Using the following translation key, the English sentence “**It is physical**” translates as “**Gx**”.

G: __ is physical

- **Universal quantifier:** the universal symbol “**∀**” followed by a variable. The universal quantifier “**∀x**” is the formal counterpart of the English words: “For every object, the following holds of **it**”.

The English sentence “**Everything is physical**” – rephrased as: “For every object, the following holds of it: it is physical” – translates as “**∀x Gx**” (using the above translation key).

- **Existential quantifier:** the existential symbol “**∃**” followed by a variable. The existential quantifier “**∃x**” is the formal counterpart of the English words: “For some object, the following holds of it”.

The English sentence “**Something is physical**” – rephrased as: “For some object, the following holds of it: it is a physical object” – is translated into the formal language as “**∃x Gx**” (using the above translation key).

- The sentence “Nothing is physical” is translated as the denial of the claim that something is physical: “ $\sim \exists x Gx$ ” (using the above translation key).